UNSTEADY TEMPERATURE FIELD IN SOLID BODIES WITH VARIABLE HEAT TRANSFER COEFFICIENT*

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Recently a great deal of attention has been given to the development of methods of calculating unsteady temperature fields in solids with variable heat transfer coefficient. This problem is encountered in investigating heat transfer in a ballistic body moving through layers with changing density, in the heating of solids by hot fluctuating streams, and so on.

The mathematical problem is formulated as follows:

$$\frac{\partial \Theta (X, \mathrm{Fo})}{\partial \mathrm{Fo}} = \frac{\partial^2 \Theta (X, \mathrm{Fo})}{\partial X^2} + \frac{k-1}{X} \frac{\partial \Theta (X, \mathrm{Fo})}{\partial X}, \quad (1)$$

$$\frac{\partial \Theta (1, \mathrm{Fo})}{\partial X} = \mathrm{Bi}(\mathrm{Fo}) [1 - \Theta(1, \mathrm{Fo})], \quad (2)$$

$$\frac{\partial \Theta (0, \mathrm{Fo})}{\partial X} = 0, \quad (3)$$

$$\Theta(X, 0) = \Theta_0. \quad (4)$$

The coefficient k in rectangular, cylindrical, and spherical coordinate systems is equal to 1, 2, and 3, respectively.

The exact solution of the problem (1)-(4) for an arbitrary dependence Bi = Bi(Fo) cannot be obtained, and only various approximate methods of evaluating the temperature field have been developed. Thus, in [2] the system (1)-(4) is reduced to a Volterra integral equation of the first kind, and in [3]—to an integral Fredholm equation of the second kind; the equations are then solved approximately.

In an investigation in [4] to determine the desired temperature, the Karman-Polhausen method from boundary layer theory is applied; the authors of [5] used a small parameter method; the solution in [6] is based on the variational principle, and so on.

Reference [1] describes a simple and quite accurate approximate method of calculating the unsteady temperature field in not too "massive" solids.

The present paper indicates a further development of this method with relevance to the heating of more "massive" bodies, characterized by increased values of the Biot number.

Using the substitution

$$W(X, \mathrm{Fo}) = \ln [1 - \Theta(X, \mathrm{Fo})] - X^n \int_0^{\mathrm{Fo}} \mathrm{Bi}^2 (\mathrm{Fo}^*)\, d\,\mathrm{Fo}^*, \quad (5)$$

in which n is a real positive number, we write the system of equations (1)-(4) in the new variable W(X, Fo):

$$\frac{\partial W}{\partial \mathrm{Fo}} = \frac{\partial^2 W}{\partial X^2} + \frac{k-1}{X} \frac{\partial W}{\partial X} +$$

$$+ n(n+k-2) X^{n-2} \int_0^{\mathrm{Fo}} \mathrm{Bi}^2(\mathrm{Fo}^*)\, d\,\mathrm{Fo}^* + f(X, \mathrm{Fo}), \quad (6)$$

$$f(X, \mathrm{Fo}) = \left[\frac{\partial \Theta (X, \mathrm{Fo})/\partial X}{1 - \Theta(X, \mathrm{Fo})}\right]^2 - X^n \mathrm{Bi}^2(\mathrm{Fo}), \quad (7)$$

$$\frac{\partial W(1, \mathrm{Fo})}{\partial X} = -\mathrm{Bi}(\mathrm{Fo}) - n \int_0^{\mathrm{Fo}} \mathrm{Bi}^2(\mathrm{Fo}^*)\, d\,\mathrm{Fo}^*, \quad (8)$$

$$\frac{\partial W(0, \mathrm{Fo})}{\partial X} = 0, \quad (9)$$

$$W(X, 0) = \ln(1 - \Theta_0) = W_0. \quad (10)$$

The nonlinear complex (7) corresponds as regards physical meaning to an internal heat source of variable intensity. It is not difficult to see that $f(X; \mathrm{Fo}) = 0$ at X = 0 [condition (3)] and at X = 1 [condition (2)]. Putting $f(X, \mathrm{Fo}) = 0$ throughout the whole heating process, and solving the system (6)-(10) [7], taking (5) into account, we find the desired temperature distribution $\Theta(X, \mathrm{Fo})$.

Values of $\Theta(1, \mathrm{Fo})$ for an Infinite Plane in a Heating Process when $\Theta_0 = 0.2$, n = 2

Fo	Θ(1, Fo)		δ, %
	according to (11)	according to computer data	
0.25	0.5312	0.5331	−0.36
0.50	0.6536	0.6570	−0.52
0.80	0.7502	0.7554	−0.69
1.00	0.7970	0.8037	−0.83
1.40	0.8534	0.8729	−2.23
2.00	0.8963	0.9332	−3.95
3.00	0.9352	0.9772	−4.30

For the infinite plate

$$\Theta = 1 - \exp\left\{\ln(1-\Theta_0) + X^n \int_0^{\mathrm{Fo}} \mathrm{Bi}^2(\mathrm{Fo}^*)\, d\,\mathrm{Fo}^* - \int_0^{\mathrm{Fo}} \mathrm{Bi}(\mathrm{Fo}^*)\, d\,\mathrm{Fo}^* - \right.$$

$$- 2 \sum_{m=1}^{\infty} \cos \mu_m X \int_0^{\mathrm{Fo}} \left\{(-1)^m \left[\mathrm{Bi}(\mathrm{Fo}^*) + n \int_0^{\mathrm{Fo}^*} \mathrm{Bi}^2(\mathrm{Fo})\, d\,\mathrm{Fo}\right] - \right.$$

$$- n(n-1) \int_0^{\mathrm{Fo}^*} \mathrm{Bi}^2(\mathrm{Fo})\, d\,\mathrm{Fo} \times$$

$$\left.\left. \times \int_0^1 X^{n-2} \cos \mu_m X dX\right\} \exp[-\mu_m^2 (\mathrm{Fo} - \mathrm{Fo}^*)]\, d\,\mathrm{Fo}^*\right\} \quad (11)$$

For the infinite cylinder

$$\Theta = 1 - \exp\left\{\ln(1-\Theta_0) + X^n \int_0^{\mathrm{Fo}} \mathrm{Bi}^2(\mathrm{Fo}^*)\, d\,\mathrm{Fo}^* - 2\int_0^{\mathrm{Fo}} \mathrm{Bi}(\mathrm{Fo}^*)\, d\,\mathrm{Fo}^* + \right.$$

$$+ 2 \sum_{m=1}^{\infty} \frac{J_0(\mu_m X)}{J_0^2(\mu_m)} \int_0^{\mathrm{Fo}} \left\{J_0(\mu_m) \left[-\mathrm{Bi}(\mathrm{Fo}^*) - n \int_0^{\mathrm{Fo}^*} \mathrm{Bi}^2(\mathrm{Fo})\, d\,\mathrm{Fo}\right] + \right.$$

$$+ n^2 \int_0^{\mathrm{Fo}^*} \mathrm{Bi}^2(\mathrm{Fo})\, d\,\mathrm{Fo} \times$$

$$\left.\left. \times \int_0^1 X^{n-1} J_0(\mu_m X)\, dX\right\} \exp[-\mu_m^2 (\mathrm{Fo} - \mathrm{Fo}^*)]\, d\,\mathrm{Fo}^*\right\}. \quad (12)$$

For the sphere

$$\Theta = 1 - \exp\left\{\ln(1-\Theta_0) + X^n \int_0^{\mathrm{Fo}} \mathrm{Bi}^2(\mathrm{Fo}^*)\, d\,\mathrm{Fo}^* - 3\int_0^{\mathrm{Fo}} \mathrm{Bi}(\mathrm{Fo}^*)\, d\,\mathrm{Fo}^* + \right.$$

$$+ 2 \sum_{m=1}^{\infty} \frac{\sin \mu_m X}{X \cdot \sin^2 \mu_m} \cdot \int_0^{\mathrm{Fo}} \left\{\sin \mu_m \left[-\mathrm{Bi}(\mathrm{Fo}^*) - n \int_0^{\mathrm{Fo}^*} \mathrm{Bi}^2(\mathrm{Fo})\, d\,\mathrm{Fo}\right] + \right.$$

$$+ n(n+1) \int_0^{\mathrm{Fo}^*} \mathrm{Bi}^2(\mathrm{Fo})\, d\,\mathrm{Fo} \times$$

$$\left.\left. \times \int_0^1 X^{n-1} \sin \mu_m X dX\right\} \exp[-\mu_m^2 (\mathrm{Fo} - \mathrm{Fo}^*)]\, d\,\mathrm{Fo}^*\right\}. \quad (13)$$

*The present paper is a continuation of the article by the authors of [1] concerning the temperature field in solid bodies with variable convective heat transfer coefficient.

The solutions obtained for $\Theta(X, Fo)$ strictly satisfy all the boundary conditions (2)–(4) and Eq. (1) at the points $X = 0$ and $X = 1$.

Numerical integration of the system (1)–(4) on a computer, when $\Theta_0 \leq 0.2$ and $n = 2$, shows that, for the entire region of variation of the argument X when $Bi \leq 3.0$ (the infinite plate), $Bi \leq 3.26$ (the infinite cylinder), and $Bi \leq 3.5$ (the sphere), the error in calculation according to (11)–(13) does not exceed 6.0%. Then the minimum error is obtained in calculating the surface temperature $\Theta(1, Fo)$, this being the item of greatest practical interest. With decrease of Bi number and increase of $|\Theta_0|$ the accuracy of the calculation is increased.

As an example we have examined the heating of an infinite plate, when the Bi number varies according to the law $Bi(Fo) = 3.0 - 2.5 \exp(-Fo)$. The table presents calculations of $\Theta(1, Fo)$ for various values of the Fourier number.

The relations (11)–(13) put forward in this paper may be recommended for approximate evaluation of the temperature field, when $Bi = Bi(Fo)$.

It should be noted that the calculation formulas may be applied also for a wider range of variation of the parameter Bi(Fo), if the exponent n is varied.

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